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Hybrid robotic system featuring a combination of a continuum robot and a rigid robotic arm: Static and differential kinematic modeling

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Abstract

This work introduces the forward instantaneous kinematics of a hybrid robotic system consisting of a compact flexible robotic instrument attached to a larger and rigid general-purpose robotic arm. Such hybrid system is expected to be of great use in robotic surgical applications where the robotic arm allows a large workspace and where the flexible robot provides additional dexterity while increasing operational safety [4]. Despite its large application potential few efforts have been conducted to rigorously model the kinematics of such hybrid systems. The emphasis of this paper is therefore on the calculation of the continuum robot's forward and differential kinematics. While the rigid robot follows traditional robotics approaches, the flexible robotic instrument, which is a cable actuated continuum robot, is modeled using the Crosserat rod theory. The proposed modeling approach describes both systems through respective manipulator Jacobian matrices and combines them into a uniform formulation of the hybrid robotic system. The method is exemplified on a rigid robotic arm featuring three rotational degrees of freedom and a flexible instrument actuated by two push-pull cable pairs. The total system thus counts five controlled degrees of freedom.

Keywords: *hybrid robot, continuum robot, kinematics, Jacobian, flexible robot*

1 Introduction

Since its first appearance in the 1920 [11], the term "robot" acquired a variety of interpretations. Humanoid machines with arms and legs mimicking the behavior of humans and animals, taking over our daily routines is an image many have carved as a sole meaning of the word robot. Over time as the robots have been developed and applied in variety of fields various robotic mechanisms have been designed. Depending on their common features and area of applications, some classifications of robotic systems have been made. This includes serial and parallel robotic mechanisms, mobile robots, aerial robots etc. One interesting classification of serial link robots which has an important aspect in this paper is a class of robots with kinematic redundancies. These robots feature more joints than it is required to complete a task. A step further is a class of hyperredundant robots. Although the difference between the kinematically redundant and hyperredundant robot is somewhat vague, a robot is considered hyperredundant if its controllable configuration space degrees of freedom are comparable to, or exceed its task space degrees of freedom [9]. Until today a variety of rigid link hyperredundant robots has been developed with a general trend showing a continuous shrinking of the link size. At its extreme, a class of hyperredundant robots where a number of joints tends to infinity and the size of joints to zero, lays a class of the so called continuum robots. A term first coined in [10], has showed an increasing appearance in the last decade. With its elastic continuous backbone a continuum robot features an infinite number of degrees of freedom. Continuum robots are both hyperredundant and underactuated.

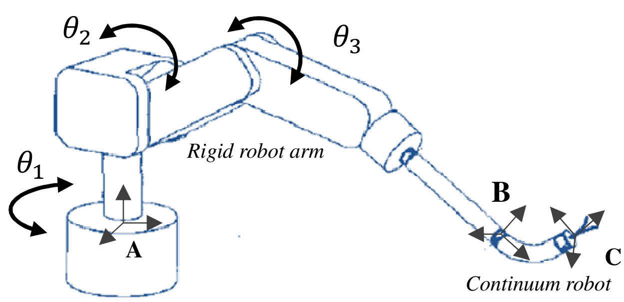

Figure 1. A flexible tool mounted on a robotic arm featuring three degrees of freedom. Flexible robotic tool has two pairs of cables for two additional degrees of freedom. Three frames are indicated on the figure. Frame **A** is the base frame of the robot. Frame **B** indicates is the base of the flexible tool. **C** is the tool frame of the robot.

The ability of continuum robots to bend and flex and conform to the difficult workspace requirements has proven its use in the field of surgical robotics. Examples of prominent continuum robotic systems can be found in the area of cardiovascular surgery [5]. The inherent compliance of continuum robots contributes to the safety of the robotic surgery in cases with reduced sensory and visual information. These characteristics could prove as valuable additions to classical endoscopic surgical robots where rigid robotic tools are traditionally used. In this way we could imagine a robotic system which features both a rigid robot arm and a continuum robotic tool as illustrated on figure 1. Here the rigid arm has three rotational degrees of freedom and serves only as an example of a rigid robotic system. Same rationale could be used for any rigid robotic system, provided that it can be described by a manipulator Jacobian.

The emphasis in this paper will be given on the calculation of the continuum robot kinematics. After exploring the initial idea of how the rigid robotic arm and a continuum robot can be combined together in section 2, a method of devising forward and differential kinematics of a continuum robot will be presented in section 3.

2 Hybrid robotic system

For rigid robots where the forward kinematics of the robot is given as a closed form expression, differential kinematics map can be derived in a straight-forward manner by derivation of the kinematic equations with respect to the joint variables [6]. The resulting matrix of partial derivatives is still a function of the joint variables and can be obtained exactly. For our case of a rigid robotic arm with three rotational degrees of freedom and the joint variables being the angles $\theta_1, \theta_2, \theta_3$ this becomes:

$$J_{AB}(\theta_1,\theta_2,\theta_3) = \left[\left(\frac{\partial g_{AB}}{\partial \theta_1} \cdot g_{AC}^{-1} \right)^{\vee} \quad \left(\frac{\partial g_{AB}}{\partial \theta_2} \cdot g_{AC}^{-1} \right)^{\vee} \quad \left(\frac{\partial g_{AB}}{\partial \theta_3} \cdot g_{AC}^{-1} \right)^{\vee} \right] \tag{1}$$

The matrix $J_{AB}(q) \in \mathbb{R}^{6x3}$ is the manipulator Jacobian of the rigid robot and $g_{AB} \in SE(3)$ the matrix of homogeneous transformation. At each configuration $(\theta_1, \theta_2, \theta_3)$ manipulator Jacobian maps the joint velocity vector into the corresponding velocity of the end-effector at frame B:

$$\mathbf{V}_{AB} = J_{AB}(\theta_1,\theta_2,\theta_3) \cdot \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T \tag{2}$$

with $V_{AB} \in \mathbb{R}^{6x1}$ being the twist of the frame B expressed in the frame A. Forward kinematics of a rigid robot is defined solely as a geometric relation. However, unlike kinematics for rigid-link robots where the pose of any point on the robot can be defined in a closed form, continuum robots are inherently elastic devices. They bend both due to the forces exerted by their internal actuators and the external environment. For continuum robots the kinematic map is changed when external forces are exerted on the robot, and therefore includes also consideration of the material properties of the robot in the calculation of the robots kinematics. The external forces are therefore also treated as a part of the configuration when considering the calculation of the robot kinematics. The manipulator Jacobian of our continuum robot $J_{BC} \in \mathbb{R}^{6x2}$ is determined by the paired cable actuation forces F_{T1} and F_{T2} and the external force F_E acting at the tip of the flexible instrument:

$$J_{BC}(F_{T1}, F_{T2}, \mathbf{F_E}) = \left[\left(\frac{\partial g_{BC}}{\partial F_{T1}} \cdot g_{BC}^{-1} \right)^{\vee} \quad \left(\frac{\partial g_{BC}}{\partial F_{T2}} \cdot g_{BC}^{-1} \right)^{\vee} \right] \tag{3}$$

$g_{BC} \in SE(3)$ is here the matrix of homogeneous transformation defining the forward kinematics of our continuum robot. As it has been previously stated, the external forces are changing the kinematic map of the continuum robot. Under the influence of the external forces a robot position is going to change according to the compliance $C_{BC} \in \mathbb{R}^{6x3}$ of the robot tip. The full term indicating the spatial velocity of a continuum robot can then be expressed considering the robot Jacobian and robot Compliance:

$$V_{BC} = J_{BC}(F_{T1}, F_{T2}, \mathbf{F_E}) \cdot \begin{bmatrix} \dot{F}_{T1} \\ \dot{F}_{T1} \end{bmatrix} + C_{BC}(F_{T1}, F_{T2}, \mathbf{F_E}) \cdot \dot{\mathbf{F}}_{\mathbf{E}} \tag{4}$$

Calculation of the manipulator Compliance matrix is not going to be considered in this paper although the same rationale employed in obtaining the manipulator Jacobian matrix can be used to obtain the manipulator Compliance matrix. Indeed, although quite different in nature, both robotic systems can be described in a similar manner if a robot manipulator Jacobian is to be derived for both robots. Then, by the virtue of coordinate transformation for spatial velocities given in different frames, it is possible by using the adjoint transformation, to express the spatial velocity of the frame C with respect to frame A.

$$\begin{aligned} \mathbf{V}_{\mathbf{AC}} &= [J_{AB} \left| Ad_{g_{AB}} \cdot J_{BC} \right] \cdot \dot{\mathbf{q}} + \left[Ad_{g_{AB}} \cdot C_{BC} \right] \cdot \dot{\mathbf{F}}_{\mathbf{E}} \\ \mathbf{q} &= \left[\theta_1 \quad \theta_2 \quad \theta_3 \quad F_{T1} \quad F_{T2} \right]^T \end{aligned} \tag{5}$$

Here Ad_{gAB} is the adjoint transformation of the rigid robot and q is the vector of joint variables of our hybrid robotic system. Clearly, traditional methodology of modeling the kinematics of a rigid link robot cannot be directly applied in the field of continuum robots. Various approaches have been considered in the modeling of the continuum robots. The earlier models have treated the robots as flexible with respect to internal actuation variables, but rigid with respect to the external forces and moments, inherently raising the constant curvature assumption as a ground model of the robot [5]. The necessity to include external forces in the kinematic modeling has been always in the perspective of various researchers, but the complexity of problems has hindered the attempts to use these types of models in real time applications. Second order differential equations with distributed boundary conditions which are often encountered when modeling continuum robots as the Cosserat rod [1] are attracting a lot of attention by various researchers. Solving and finding the Jacobian matrix for continuum robots is challenging due to the complexity of the underlying kinematic model. The most significant work in finding exact equations to calculate the Jacobian is the work of Rucker et al. [3]. In this work after the forward kinematics of the continuum robot is obtained by solving a boundary value problem, a new set of differential equations defining the Jacobian matrix of the robot is solved using the initial values obtained after the solution of the forward kinematics problem. The novelty of our work lies in the description of the forward and differential kinematics in a form posed as an initial value problem. In that case both the forward and the differential kinematics are obtained simultaneously in one integration step starting from the tip of the robot to the base of the robot.

3 Continuum robot kinematics

The general idea of modeling continuum robots under large deformations lies in parameterization of the robots centerline by arc length. It is assumed that the entire deformation of our robot lies in the curve described by the robot centerline. Then, for each arc length coordinate starting from $s = 0$ at the frame B to $s = L$ at the robots tool frame C, there exists a coordinate system attached to the robot centerline $X(s)$, called the body frame. Figure 2 illustrates this principle.

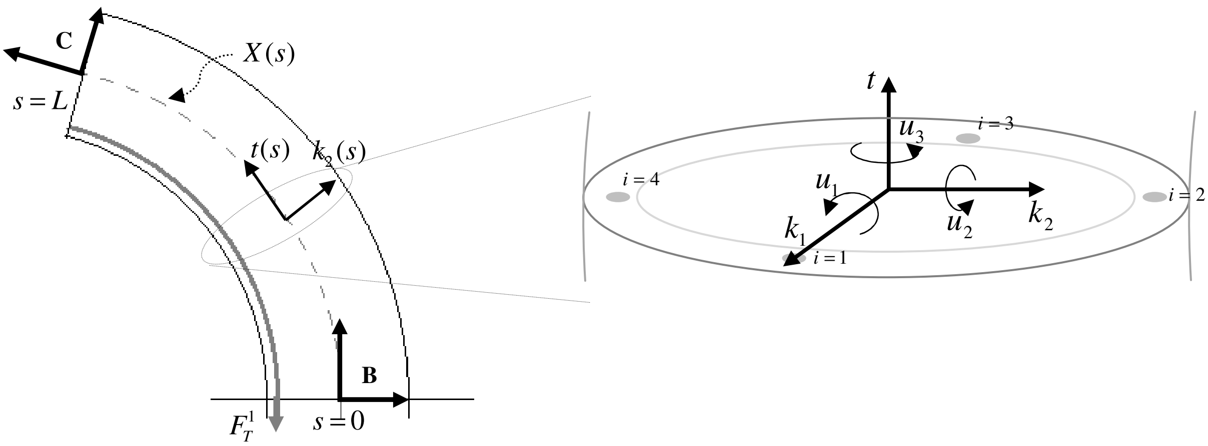

Figure 2. Our continuum robot consists of a super-elastic structure with four embedded cables (labeled with i = 1 .. 4). At every cross section of the robot, at arc length coordinate s, a frame is attached which coincides with the material basis of the robot. Curvature $u_1(s)$ and $u_2(s)$ correspond to the bending of the robot and $u_3(s)$ relates to torsion of the robot.

The deformation of the robot happens under the influence of various forces acting on the robot backbone. These forces can be internal actuation forces or external environmental forces. The actuation forces are exerted by robots internal driving mechanism which can be McKibben mucles [7], cables [5] or even internal intrinsic curvatures [2], [8]. The environmental forces are all the remaining forces acting upon the structure. This can be gravity as well as the contact forces between the robot and the environment. A behavior of such an elastic structure subjected to large deformations can be described by Cosserat rod model [1]. Written in the body frame as observed on the figure 2, the governing equilibrium equations of the continuum robot can be expressed in a form:

$$\frac{d\mathbf{m}(s)}{ds} = -\hat{u}(s)\cdot\mathbf{m}(s) - \hat{v}(s)\cdot\mathbf{n}(s) + \mathbf{l}(s)$$
$$\frac{d\mathbf{n}(s)}{ds} = -\hat{u}(s)\cdot\mathbf{n}(s) + \mathbf{f}(s) \tag{6}$$

These arc-length parameterized equations with $m(s) \in \mathbb{R}^3$ and $n(s) \in \mathbb{R}^3$ being moment and shear forces acting at the arc length coordinate s are defining the static equilibrium equation of the rod under deformation. $l(s) \in \mathbb{R}^3$ and $f(s) \in \mathbb{R}^3$ are the distributed moment and forces acting along the robot. In case of a cable driven design which is analyzed in this paper, the distributed moment and forces are raised by continuous interaction between the cable and the elastic structure of the robot. $\hat{u}(s)$ and $\hat{v}(s)$ indicate skew symmetric matrices of angular and linear strains:

$$\hat{u}(s) = \begin{bmatrix} 0 & -u_3(s) & u_2(s) \\ u_3(s) & 0 & -u_1(s) \\ -u_2(s) & u_1(s) & 0 \end{bmatrix}, \hat{v}(s) = \begin{bmatrix} 0 & -v_3(s) & v_2(s) \\ v_3(s) & 0 & -v_1(s) \\ -v_2(s) & v_1(s) & 0 \end{bmatrix} \tag{7}$$

3.1 Forward kinematics

The static equilibrium equations (6) are capturing the behavior of an elastic rod subjected to large deformations. They relate the forces, moments and curvatures for a general type of a robot. To account for a specific continuum robot design, these equations need to be adapted. For a cable driven continuum robot illustrated in the figure 2, the effect of cables is considered when observing the distributed moment and force equations on the backbone of the robot:

$$\mathbf{l}(s) = \mathbf{e_3}\cdot d\cdot\left[\left(F_T^1 - F_T^3\right)\cdot u_1(s) - \left(F_T^4 - F_T^2\right)\cdot u_2(s)\right]$$
$$\mathbf{f}(s) = \sum_i F_T^i\cdot\left[u_2(s) \quad -u_1(s) \quad 0\right]^T \tag{8}$$

The right superscripts i for tension forces F_T^i indicates the position of the cable inside the robot. The distance d of the cable to the centerline of the robot is constant for each arc length coordinate s. e_3 is a unit vector of the body frame coordinate system in the direction of the t axis, $e_3 = [0 \quad 0 \quad 1]^T$. It can be observed that the cables at position $i = 1$ and $i = 3$ as well as cables at $i = 2$ and $i = 4$ form a two pair of cables as their mutual effects cancel out. Since the cables can only experience tension this allows us to replace the cable actuation forces F_T^i with two variables which can be both positive and negative. Further in the text these two variables will be regarded as cable tension forces or joint variables:

$$F_{T1} = F_T^1 - F_T^3$$
$$F_{T2} = F_T^4 - F_T^2 \tag{9}$$

It should be noted that the distributed force term does not include the contribution of the gravitational forces, indicating that our continuum robot has a negligible mass. Another simplification to the model can be taken for a robot which has a negligible shear in comparison to the angular bending. Then the linear strains can be fixed to:

$$v(s) = e_3 = \left[0 \quad 0 \quad 1\right]^T \tag{10}$$

Combining the equations (6), (8), (9) and (10) we arrive at moment and force equilibrium equations specific to our robot:

$$\frac{d\mathbf{m}(s)}{ds} = -\hat{u}(s)\cdot\mathbf{m}(s) - \hat{e}_3\cdot\mathbf{n}(s) - \mathbf{e_3}\cdot d\cdot\left[F_{T1}\cdot u_1(s) - F_{T2}\cdot u_2(s)\right]$$
$$\frac{d\mathbf{n}(s)}{ds} = -\hat{u}(s)\cdot\mathbf{n}(s) - \sum_i F_T^i\cdot\left[u_2(s) \quad -u_1(s) \quad 0\right]^T \tag{11}$$

Additional term is required to relate moments and forces to curvatures. This term would define the material model of our robot and it would be governed by the materials and material properties of the robot structure. If a robot is assumed to be perfectly elastic and homogenous along the entire backbone, Hooks law could be used to relate the moments and curvatures of the robot:

$$\mathbf{m}(s) = \begin{bmatrix} K_b & 0 & 0 \\ 0 & K_b & 0 \\ 0 & 0 & K_t \end{bmatrix} \cdot \mathbf{u}(s) = K \cdot \mathbf{u}(s) \tag{12}$$

K_B is the bending stiffness of the robot and K_T the torsional stiffness of the robot. They form the elements of the stiffness matrix K. Matrix K is hereby diagonal indicating that our body frame coincides with the material basis of the robot for every arc length coordinate s. Equations (11) and (12) are specific to our robot. They contain the description of our robot with respect to the actuation mechanism and the way how the actuators exert forces on the robots elastic structure as well as the material description of our robot. Following the Cosserat rod theory additional expression can be derived relating the curvature $u(s)$ and the rotation matrix $R(s)$. According to Antmann [antmann], the equations describing the dynamics of a tumbling rigid body are identical in form to those describing the equilibrium of a rod with linear constitutive relations, therefore giving a relation between the matrix of rotation $R(s)$ and the curvatures $u(s)$:

$$\frac{dR(s)}{ds} = R(s) \cdot \hat{u}(s) \tag{13}$$

In a similar way for the rate of change of the position of the centerline along the arc length, a relation is given to the rotation and linear strains, which in our case of negligible shear strains simplifies to:

$$\frac{d\mathbf{X}(s)}{ds} = R(s) \cdot \mathbf{v}(s) = R(s) \cdot \mathbf{e_3} \tag{14}$$

The equations (11)-(14) represent a full set of differential equations which defines the equilibrium state of our continuum robot. To solve the equations a set of boundary conditions are required. First set of boundary conditions can be given for the force moment equilibrium equations. Assuming that the contact of the robot with the environment happens only at the tip of the robot where environment interaction force F_E and the cable forces F_T^1-F_T^4 are exerted, the boundary condition for force equations are:

$$\mathbf{n}(L) = \mathbf{F_E} - \sum_i F_T^i \cdot \mathbf{e_3} \tag{15}$$

The moment equilibrium equations are determined only by the cable tension forces F_{T1} and F_{T2} if the environment force F_E is assumed to act only at the center of the robot tip:

$$\mathbf{m}(L) = d \cdot \begin{bmatrix} F_{T2} \\ F_{T1} \\ 0 \end{bmatrix} \tag{16}$$

Both the cable termination forces and the environment interaction force F_E are given in the body coordinate frame of the robot tip. Since our equations are given in a form independent of a fixed coordinate frame it is convenient to choose the boundary conditions for rotation $R(s)$ and the position $X(s)$ to coincide with the body coordinate frame of the robot C. Hence, we can set the rotation matrix to equal the unit matrix, indicating no rotation:

$$R(L) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

And have for the origin of the frame C to coincide with the body frame at the tip of the robot:

$$\mathbf{X}(L) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{18}$$

The forward kinematics and the pose of the continuum robot are then obtained by integration from the tip of the robot at frame C where $s = L$ to the base of the continuum robot and frame B where $s = 0$. The solution of the system of equations at $s = 0$ for $X(0)$ and $R(0)$ provides the description of the forward kinematics of the robot and a matrix of homogeneous transformation can be constructed as:

$$g_{CB} = g(0) = \begin{bmatrix} R(0) & X(0)^T \\ 0 & 1 \end{bmatrix} \tag{19}$$

The matrix of homogenous transformation g_{CB} gives the relation of the coordinate frame B with respect to the coordinate frame C. However, since we are interested in obtaining the relation of the frame C with respect to the frame B which also represents a physical constraint of the robot, we can look at the inverse of the homogeneous transformation of g_{CB} and arrive at:

$$g_{BC} = g^{-1}(0) \tag{20}$$

3.2 Differential kinematics

To obtain the manipulator Jacobian of a continuum robot, the equations of forward kinematics are derived with respect to the continuum robots joint variables. This raises a new set of differential equation albeit larger than the first set of equations which define only the forward kinematics of the robot. Following the same rationale employed in section 3.1, the manipulator Jacobian is computed by integration from the tip of the robot to the base of the robot, following the same rationale employed in forward kinematics.

Starting off with the first set of moment and force equilibrium equations given in a general form (7) we can take their partial derivatives with respect to the actuator forces F_{Ti}:

$$\begin{aligned} \frac{d}{ds}\left(\frac{\partial \mathbf{m}}{\partial F_{Ti}}\right) &= -\frac{\partial \hat{u}}{\partial F_{Ti}} \cdot \mathbf{m} - \hat{u} \cdot \frac{\partial \mathbf{m}}{\partial F_{Ti}} - \frac{\partial \hat{v}}{\partial F_{Ti}} \cdot \mathbf{n} - \hat{v} \cdot \frac{\partial \mathbf{n}}{\partial F_{Ti}} + \frac{\partial \mathbf{l}}{\partial F_{Ti}} \\ \frac{d}{ds}\left(\frac{\partial \mathbf{n}}{\partial F_{Ti}}\right) &= -\frac{\partial \hat{u}}{\partial F_{Ti}} \cdot \mathbf{n} - \hat{u} \cdot \frac{\partial \mathbf{n}}{\partial F_{Ti}} + \frac{\partial \mathbf{f}}{\partial F_{Ti}} \end{aligned} \tag{21}$$

A new set of equations is hereby obtained, where all the variables which appear in the equation (21) and are partial derivatives with respect to the actuators forces can be considered as unknowns and independent variables themselves. When observing these equations we can note the terms of moment, force and strain matrices also appear inside the equation. This means that the equations defining the differential kinematics of our continuum robot are coupled to the solution of the forward kinematics. This however is not a problem if the forward kinematics and the differential kinematics are solved simultaneously. Differentiating the material model of our robot and the equation (12) we obtain:

$$\frac{\partial \mathbf{m}}{\partial F_{Ti}} = K \cdot \frac{\partial \mathbf{u}}{\partial F_{Ti}} \tag{22}$$

For equation (12) differentiation results in:

$$\frac{d}{ds}\left(\frac{dR}{\partial F_{Ti}}\right) = \frac{\partial R}{\partial F_{Ti}} \cdot \hat{u} + R \cdot \frac{\partial \hat{u}}{\partial F_{Ti}} \tag{23}$$

And finally after differentiating (13) the partial derivatives of the position with respect to the joint variables become:

$$\frac{d}{ds}\left(\frac{d\mathbf{X}}{\partial F_{Ti}}\right) = \frac{\partial R}{\partial F_{Ti}} \cdot \mathbf{e_3} \tag{24}$$

In this way the equations defining the forward kinematics have been transformed into a new set of equations (21)-(24) which describe the differential kinematics of the robot. To solve the system of these equations a set of boundary conditions is again required. The boundary conditions for this can be obtained by taking the partial derivatives of the boundary conditions of the forward kinematics. For force boundary equations this becomes:

$$\frac{d\mathbf{n}}{\partial F_{Ti}}(L) = -e_3 \tag{24}$$

Differentiating moment boundary equation (16) for both pairs of cable actuation forces:

$$\frac{d\mathbf{m}}{\partial F_{T1}}(L) = d \cdot \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T, \frac{d\mathbf{m}}{\partial F_{T2}}(L) = d \cdot \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \tag{26}$$

And finally boundary equations for partial derivatives of rotation and position become:

$$\frac{\partial R}{\partial F_{Ti}}(L) = 0, \frac{\partial \mathbf{X}}{\partial F_{Ti}}(L) = 0 \tag{27}$$

Integrating equations (21)-(24) from the frame C with $s = L$ to the frame B with s=0 simultaneously with the equations of the forward kinematics and starting from the initial conditions given by the equations (25), (26) and (27) we will obtain the solution of our differential kinematics problem. Then, taking the solution matrix of the system for the rotation derivatives and position derivatives at $s = 0$, we can assemble the partial derivative matrix of homogeneous transformation:

$$\frac{\partial g_{CB}}{\partial F_{Ti}} = \frac{\partial g}{\partial F_{Ti}}(0) = \begin{bmatrix} \frac{\partial R}{\partial F_{Ti}}(0) & \frac{\partial X}{\partial F_{Ti}}(0)^T \\ 0 & 0 \end{bmatrix} \tag{28}$$

And construct the manipulator Jacobian matrix of the continuum robot in analogy to the traditional rigid arm:

$$J_{CB} = \begin{bmatrix} \frac{\partial g_{CB}}{\partial F_{T1}} \cdot g_{CB}^{-1} & \frac{\partial g_{CB}}{\partial F_{T2}} \cdot g_{CB}^{-1} \end{bmatrix} \tag{29}$$

The assembled Jacobian matrix is observed in the frame B and with respect to frame C. Again we are interested in the representation of the Jacobian matrix in the frame B. This can be achieved by application of the adjoint transformation:

$$J_{BC} = -Ad_{g_{CB}} \cdot J_{CB} \tag{30}$$

4 Conclusion

In this paper we have presented a methodology of devising the forward and differential kinematics for a continuum robot which allows for an efficient computation of the kinematic equations in one integration step. In the calculation of the kinematics of the continuum robot, the elasticity of the robot as well as the interaction forces with the environment are taken into account. Placed in the general framework of the robotics and modeled by a manipulator Jacobian matrix, a continuum robot can be seamlessly integrated with a rigid robotic arm.

The existence of the environment interaction force which deforms the continuum robot here is regarded as a part of the robots configuration. This indicates a necessity of a force sensor in our robotic system. The current formulation expressed in this paper assumes that the interaction force is known and measured at the tip of the flexible surgical instrument. Another possibility is to measure the force at the base of the flexible robot and frame B. This would also provide us with a set of boundary conditions for the solution of robot kinematics.

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